Final Test
Motion and Manipulation

November 8, 2017
13:30-16:00

Note: It is not allowed to use pocket calculators or consult books, notes, slides, etc. Fill out your name and student number on each page you hand in. The test consists of **nine** exercises. **Motivate all your answers!**

1: Kinematics I (1.0)

We are given a fixed orthonormal frame $F = \{f^1, f^2, f^3\}$ and a mobile orthornormal frame $M = \{m^1, m^2, m^3\}$. Initially the frames M and F coincide. We translate M along f^1 by 2 units and then rotate M about m^2 by $\pi/4$ radians. Determine the homogeneous transformation matrix that maps mobile M coordinates into fixed F coordinates. Transform the M coordinates $(1, 0, 1)$ into F coordinates.

2: Kinematics II (1.0)

Use quaternions to determine the image of the point $p = (2, 0, 0)$ after a rotation by an angle of $\pi/2$ about the line through the origin with direction vector $(1, -1, 0)^T$.

3: Kinematics for Linkages (1.0)

Consider the four-axis arm on the separate sheet and the frames assigned to its axes of motion and its hand. Use the given frames to determine the joint angle θ_i, the joint distance d_i, the link length a_i, and the link twist angle α_i for each of the axes $i = 1, 2, 3, 4$. Clearly indicate which parameters are variables.

4: Inverse Kinematics (0.25+0.25+1.5)

Consider a rotating variable-length line segment S of which one endpoint is fixed at the origin O of the standard two-dimensional coordinate frame. We refer to the other (free) endpoint of S as the tip. The segment can rotate about O and simultaneously vary its length. Let l be the length of S and let θ be its orientation (given by the counterclockwise angle with the positive x-axis). The relation between the position (x, y) of the tip of S and the joint variables l and θ is now given by the simple relation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} l\cos\theta \\ l\sin\theta \end{pmatrix}.$$

(a.) Explain why this relation holds.

Consider the inverse kinematics problem of finding the values of l and θ that place the tip at the point $(1, 1)$.

(b.) Show that $l = \sqrt{2}$ and $\theta = \pi/4$ place the tip of S at $(1, 1)$.

Although it turns out to be easy to determine an analytic solution to our inverse kinematics problem we consider the use of the iterative solution method.

(c.) Perform one iteration of the iterative solution method to find improved values $l^{(1)}$ and $\theta^{(1)}$ for the joint variables l and θ respectively, using initial values $l^{(0)} = 2$ and $\theta^{(0)} = 0$.

5: Minkowski Sums (1.0)

Let s_0 be the line segment with endpoints $(1,2)$ and $(3,1)$ and let s_1 be the line segment with endpoints $(1,2)$ and $(2,4)$. Let $L = s_0 \cup s_1$. Let t be the segment with endpoints $(-3,-2)$ and $(-5,-4)$. Construct the Minkowski sum $L \oplus t$. Is $L \oplus t$ equal to $t \oplus L$?

6: Short Questions (0.5+0.5)

(a.) Name one advantage of the use of oriented bounding boxes (OBBs) over spheres as bounding volumes in a bounding volume hierarchy. Also name one advantage of the use of spheres over OBBs as bounding volumes in a bounding volume hierarchy.

(b.) Determine the Plücker coordinates of the line ℓ through the points $(1,1,1)$ and $(1,3,6)$.

7: Form Closure Grasps (1.0)

The boundary of the convex semi-algebraic object

$$C = \{(x,y) \in \mathbb{R}^2 | x^2 + y^2 - 25 \leqslant 0\} \cap \{(x,y) \in \mathbb{R}^2 | -x - 1 \leqslant 0\}$$

consists of one circular arc and one line segment. Let $a_1 = (4,-3)$ and $a_2 = (3,4)$. Choose two points a_3 and a_4 along the boundary of C such that frictionless point fingers at a_1, a_2, a_3, and a_4 put C in form closure. Use Reuleaux' half-plane analysis of instantaneous velocity centers to justify your answer.

8: Force Closure Grasps (1.0)

Consider the points $p_1 = (0,0)$, $p_2 = (8,-2)$, $p_3 = (0,6)$, and $p_4 = (-8,-2)$. Let V be the object bounded by the edges p_1p_2, p_2p_3, p_3p_4, and p_4p_1. Let $a_1 = p_1 = (0,0)$ and $a_2 = (-6,0)$. Determine a point a_3 along the interior of the edge p_2p_3 such that frictionless point fingers at a_1, a_2, and a_3 do *not* put V in force clsoure. Use wrench analysis to justify your answer.

9: Manipulation (1.0)

Construct a convex polygonal object P with five vertices and choose a center of mass inside P such that P has exactly *four* stable orientations when being pushed by a jaw.