FINAL EXAM SUBATOMIC PHYSICS (Feb. 1st, 2013)

NS-369B

For this exam you are allowed to use your book (Subatomic Physics by Henley and Garcia), the notes from the first lecture about relativistic kinematics, and a simple calculator. Other materials are not allowed.

Important note: Start the solution of each exercise on a different sheet of paper. Explain your answers clearly, and don't forget to write units.

Grading: In total you can get a total of 100p on the exam. The number of points indicated on each subquestion is meant to be a good indication, however minor changes may be made.

Exercise 1: Short Questions (25p)

a) Consider a composite particle A, and (one of) its excited states, denoted by A^*. Now consider the reaction where the excited state falls to the ground state, thereby emitting a photon, i.e.: $A^* \to A+\gamma$. Is the excited state heavier or lighter than the ground state? Back your answer up with a calculation. **(10p)**

b) Consider the elastic scattering (i.e., no new particles are created) of two particles (see Fig. 1). Denote the four-momenta of the incoming particles by p_1^μ and p_2^μ, and the outgoing particles by p_3^μ and p_4^μ. For these kind of processes, one often uses the so-called Mandelstam variables, which are defined as: $s = (p_1+p_2)_\mu(p_1+p_2)^\mu$, $t = (p_1-p_3)_\mu(p_1-p_3)^\mu$, and $u = (p_1-p_4)_\mu(p_1-p_4)^\mu$. Show that $s+t+u = m_1^2+m_2^2+m_3^2+m_4^2$. **(10p)**

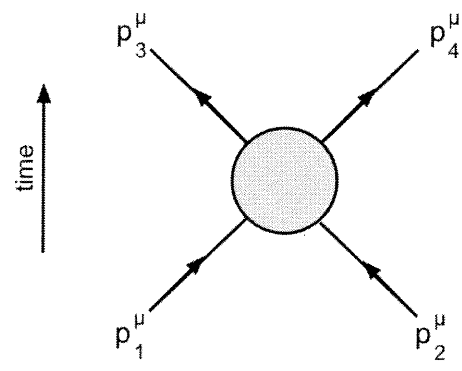

Figure 1: Elastic collision of two particles.

c) In class you have seen that theories of subatomic physics, such as QED and QCD, predict that the interaction strength between particles depends on the energy scale (or equivalently on the distance scale) at which the processes occur. Theoretically it is possible to (approximately) predict the coupling at a certain scale μ, given that we know the coupling at a certain scale μ_0. For a "QCD-like" theory with N_c colors and N_f flavors, this relation is given by:

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1+\frac{\alpha_s(\mu_0)}{6\pi}(11N_c-2N_f)\ln(\mu/\mu_0)}. \tag{1}$$

QCD has the property that it is strongly interacting at low energies, and weakly interacting at high energies. Suppose nature had a different number of flavors and/or colors. Give a condition on N_c and N_f for which the situation was reversed, and QCD would be weakly interacting at low energies, and strongly interacting at high energies. Would the world look any different? **(5p)**

Exercise 2: Baryon Ground State (35p)

Baryons are particles which consist of three quarks. Generally, in a system of three particles, there are two orbital angular momenta. In this exercise, however, we will assume that the baryon is in its ground state, and we therefore assume both orbital angular momenta to be zero. Furthermore, we will restrict ourselves to the lightest two quark flavors, up and down. Since quarks are fermions, a baryon state needs to be anti-symmetric under the exchange of any two quarks[1], i.e., if one quark were to switch places with another, the state should get a minus sign.

We start by looking at the *flavor*-part of the baryon state. Recall that the isospin state of the up-quark is $|\frac{1}{2},\frac{1}{2}\rangle$, and the isospin state of the down-quark is $|\frac{1}{2},-\frac{1}{2}\rangle$, where the quantum numbers in the ket are: $|I, I_3\rangle$. As a consequence, the total isospin of a baryon consisting of only up's and down's can be either $I = 1/2$, or $I = 3/2$.

a) Write down all the possible $I = 3/2$ isospin states in terms of u's and d's. Don't forget the normalization factor. *Hint: use the lowering operator.* **(8p)**

b) The $I = 3/2$ isospin baryons are called Δ's. Identify the states that you found in part a) with Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-}. **(2p)**

c) The Δ^+ baryon has two decay modes, namely $\Delta^+ \to p + \pi^0$, and $\Delta^+ \to n + \pi^+$. Which of the two decay modes occurs more frequently? By how much? **(10p)** *Hint: The Clebsch-Gordan coefficients for adding a (iso)spin-1 to a (iso)spin-1/2 state are given by:*

$$\begin{aligned}
|1,1\rangle\,|\frac{1}{2},\frac{1}{2}\rangle &= |\frac{3}{2},\frac{3}{2}\rangle\,, \\
|1,0\rangle\,|\frac{1}{2},\frac{1}{2}\rangle &= \sqrt{2/3}\,|\frac{3}{2},\frac{1}{2}\rangle - (1/\sqrt{3})\,|\frac{1}{2},\frac{1}{2}\rangle\,, \\
|1,-1\rangle\,|\frac{1}{2},\frac{1}{2}\rangle &= (1/\sqrt{3})\,|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{2/3}\,|\frac{1}{2},-\frac{1}{2}\rangle\,, \\
|1,1\rangle\,|\frac{1}{2},-\frac{1}{2}\rangle &= (1/\sqrt{3})\,|\frac{3}{2},\frac{1}{2}\rangle + \sqrt{2/3}\,|\frac{1}{2},\frac{1}{2}\rangle\,, \\
|1,0\rangle\,|\frac{1}{2},-\frac{1}{2}\rangle &= \sqrt{2/3}\,|\frac{3}{2},-\frac{1}{2}\rangle + (1/\sqrt{3})\,|\frac{1}{2},-\frac{1}{2}\rangle\,, \\
|1,-1\rangle\,|\frac{1}{2},-\frac{1}{2}\rangle &= |\frac{3}{2},-\frac{3}{2}\rangle\,.
\end{aligned}$$

We will now look at the *spin*-part of the baryon state. Since all quarks have spin-1/2, the total spin of a baryon can be spin-1/2, or spin-3/2. It turns out that all the Δ baryons have spin-3/2.

d) Write down all the possible spin states of a spin-3/2 baryon in terms of $\uparrow$'s and $\downarrow$'s, where $\uparrow \equiv |\frac{1}{2},\frac{1}{2}\rangle$, and $\downarrow \equiv |\frac{1}{2},-\frac{1}{2}\rangle$. **(5p)**

e) The total baryon state is now the product of the isospin part and the spin part[2], i.e., $\psi_\Delta = \psi_{\text{isospin}}\psi_{\text{spin}}$. Show that ψ_Δ is symmetric under exchange of two quarks. **(5p)**

To fix the problem with the symmetric total state, physicists came up with a new quantum number, namely "color", i.e., quarks can be red (r), green (g) or blue (b). The total baryon-state becomes: $\psi_\Delta = \psi_{\text{isospin}}\psi_{\text{spin}}\psi_{\text{color}}$, where ψ_{color} should be (totally) anti-symmetric under exchange of quarks.

f) Write down the totally anti-symmetric color state ψ_{color}. Don't forget the normalization factor. Is this state unique? **(5p)**

[1] Note that I mean really any two quarks, so also if you interchange a u-quark with a d-quark or a s-quark, your baryon state still gets a minus sign. In this sense, quarks of different flavors are really just the same particle in a different state, just like an electron with spin up is the same particle as an electron with spin down but in a different state.

[2] In principle there's also a *space*-part of the baryon state, however this part can be assumed to be symmetric under exchange of two quarks, since we're considering the ground state.

Exercise 3: Cascade Particles[3] (40p)

Many particle experiments use tracking detectors, i.e., detectors which are able to record the tracks of particles in a certain volume[4]. Examples of such detectors are wire chambers, bubble chambers, and time projection chambers, the last one being used by modern experiments such as ALICE and STAR. In collider experiments such tracking detectors are built around the collision point, in order to get a complete picture of all the tracks generated in a certain collision.

In Fig. 2 you see a schematic picture of the reconstructed tracks in a p+p collision[5]. In this exercise we only concern ourselves with the particle species mentioned in this picture. Furthermore, we assume that all particles decay as indicated in the picture, i.e., we ignore all the particle's other decay channels. The masses and lifetimes of the particles in the picture are as follows:

Particle	Mass (MeV/c^2)	Mean lifetime (sec)
$\pi^\pm$	140	2.6×10^{-8}
$K^\pm$	494	1.2×10^{-8}
Λ^0	1116	2.6×10^{-10}
Δ^0	1232	5.6×10^{-24}
Ξ^-	1321	2.9×10^{-10}
Ω^-	1672	8.2×10^{-11}

a) Assume we record a large number of $p+p$ collisions, similar to the one in Fig. 2. Make a sketch of the invariant mass distribution (i.e., the number of $p\pi^-$ pairs ($N_{p\pi^-}$) as a function of the invariant mass of that pair $m_{p\pi^-}$). *Hint: pay attention to the lifetime of the particles.* **(10p)**

b) In another figure, make a sketch of the invariant mass distribution of the $p\pi^-\pi^-$ triplets. **(5p)**

c) It occasionally happens that a kaon is misidentified by the detector as a pion. What effect does this have on the invariant mass distribution that you drew in part b)? **(5p)**

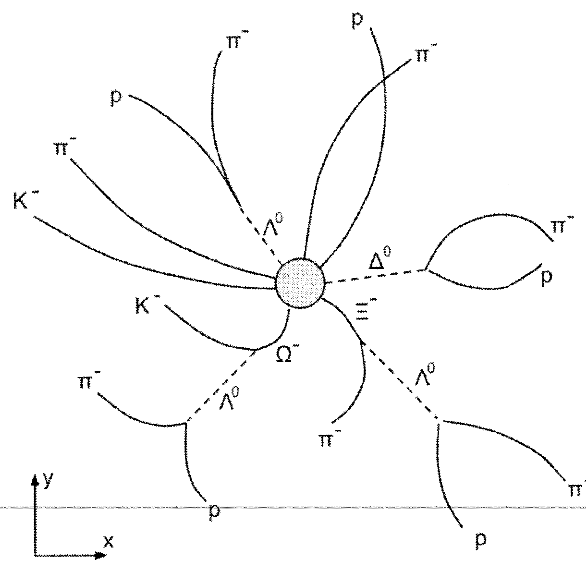

Figure 2: Schematic event display. This picture is not to scale, and the length of the tracks in this picture should not be used as an argument in your answer. Uncharged particles are not detected, and are indicated with a dotted line.

[3] Particles like Ξ and Ω are called cascade particles, since their decay products are unstable and decay again.

[4] Of course all that a tracking detector really records are "pixels" along the path of the particle, and we need reconstruction software to find which pixels together form the track of a particle. This process becomes increasingly difficult if a collision produces more particles. Another complication comes from the fact that the detector will have some noise, i.e, it records random pixels, which may mistakenly be assumed to be part of a track by the detector.

[5] This is an oversimplified picture, details which do not concern us are left out.

As you can see in Fig. 2, the tracks of the decay products of the unstable particles do not intersect with the collision point. This means that we reduce the "background" in our invariant mass plots, by throwing away tracks which do not originate from the collision point.

d) For unstable particles, we can relate the number of particles at time t to the number of particles at time 0 as follows: $N(t) = N(0)\exp\{-t/\tau\}$, where τ is the particle's lifetime[6].

Suppose now that we're dealing with a particle species with mass m, traveling with a momentum $\mathbf{p}$. Show that we can find the number of particles after traveling a distance x by **(8p)**:

$$N(x) = N(0)\exp\left\{-\frac{mx}{|\mathbf{p}|\tau_{\text{rest}}}\right\}. \tag{2}$$

e) Suppose now that we filter out all the protons and pions which originate from within 0.5cm of the collision point. What percentage of Λ^0's, Δ^0's, Ξ^-'s, and Ω^-'s will we lose by this filtering, if we make the (somewhat silly) assumption that all those baryons have a momentum of $|\mathbf{p}| = 1\text{GeV/c}$, and that their momentum is completely in the xy-plane. *Use* $c = 3 \times 10^8 m/s$. **(5p)**

f) Describe the effects that this filtering has on the invariant mass distribution of part a), and b) in words and with a sketch, and pay special attention to what happens to the background. **(7p)**

[6] The lifetime should of course be properly Lorentz-transformed if we're not in the particle's rest frame.