Prioriteit : 2 WISB 272

Deeltentamen Speltheorie 2007

(per 1/3)

07.11.07, 14-17 u., BBL 105b en 107a

(The total number of points is 100: success with scoring them all!)

1. (10 points) The game of nim starts with three heaps of 10, 28 and 31 stones. Determine if the starting position is a N-position or a P-position. If the starting position is a N-position, find all winning initial moves.

2. (15 points) The following function is the value of a 2×2 matrix game:

$$V(t) = \text{Val}\begin{pmatrix} 1 & 5 \\ 2t+1 & 2 \end{pmatrix}.$$

Draw the graph of $V(t)$ for $-\infty < t < \infty$. Do not miss possible saddle-points.

3. Find the value and optimal (that is minimax) strategies in the following games with matrices

(a) (10 points) $\begin{pmatrix} 2 & -3 & 6 & 5 \\ 1 & 1 & 3 & 2 \\ 5 & 7 & 2 & 1 \end{pmatrix}$, (b)(10 points) $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 0 & 5 \\ 0 & 5 & -2 \end{pmatrix}$,

(c) (10 points) $\begin{pmatrix} 5 & 7 & 2 & 1 \\ 3 & 2 & 4 & 6 \end{pmatrix}$.

4. (10 points) In the game with matrix $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 1 & 2 \end{pmatrix}$ both players have optimal strategies giving positive probability to each of the pure strategies. Based on this fact find the value of the game and the optimal strategy of Player 1.

5. (10 points) Suppose a game with $m \times n$ matrix $A = (a_{i,j})$ has the property that

$$a_{i,j} = a_{\pi(i),\sigma(j)} \qquad i = 1, \ldots, m; \; j = 1, \ldots, n,$$

where $\pi : \{1, \ldots, m\} \to \{1, \ldots, m\}$ and $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ are some given permutations. Argue that solving the game with matrix A can be reduced to solving a game with some $k \times \ell$ matrix B, where k is the number of cycles of π, and ℓ is the number of cycles of σ. Given optimal strategies of both players in the game with matrix B, how can you find optimal strategies in the game with matrix A?

6. Consider a combinatorial game with several heaps of stones and two distinct types of moves. In one move a player may

either remove one, two or three stones from one of the heaps,

or split one heap of size $n \geq 3$ in two heaps of sizes 2 and $n - 2$.

(a) (15 points) Determine the Sprague-Grundy function of the game. Support your answer by a complete formal proof.
(b) (10 points) Suppose the game starts with three heaps of sizes 3, 5 and 7. Determine if this initial position is a N-position and if the answer is affirmative find all winning initial moves.