- There are 4 hours available for the problems.
- Each problem is worth 10 points.
- Be clear when using a theorem. When you are using an obscure theorem, cite a source.
- Use a different sheet for each problem.
- Clearly write DRAFT on any draft page you hand in.

MOAWOA

May 13, 2016

Problem 1. An invertible 2×2-matrix M with real entries is called a *MOAWOA-matrix* if its inverse M^{-1} can be obtained by permuting the entries of M. Show that if M is a MOAWOA-matrix, then so is M^2.

Problem 2. Suppose I and J are (real) open intervals of finite positive length, each interval not containing the other. Prove that there exists a $\lambda \neq 0$ such that $x \mapsto e^{\lambda x}$ maps I and J to intervals of equal length if and only if I and J have different lengths.

Problem 3. Consider n people that stand in a circle. Initially, each of them holds a red and a blue ball. In a turn, each person gives one of his balls to his right neighbor and his other ball to his left neighbor. Does there exist a sequence of turns (starting from the initial situation) such that every possible color distribution of the balls occurs exactly once

(a) if $n = 2016$?

(b) if $n = 2015$?

Problem 4. We consider sequences $a_0, a_1, a_2, \ldots$ of real numbers that satisfy

$$a_n = 4a_{n-1}(1 - a_{n-1})$$

for all positive integers n. How many such sequences satisfy $a_{2016} = a_0$?

Problem 5. We are given N weights, with masses 1 kg, 2 kg, ..., N kg. We want to select at least two of these weights, such that their total mass equals the average mass of the other weights. Show that this is possible if and only if $N + 1$ is a square.

Problem 6. Let k be a positive integer. We consider all possible football matches in which $2k$ goals are scored in total. Prove that the number of such matches in which the end result is a draw equals the number of such matches in which the home team is never behind. (By a "football match" we mean the set of all intermediate scores that occur during the match.)