- There are 4 hours available for the problems.
- Each problem is worth 10 points.
- Be clear when using a theorem. When you are using an obscure theorem, cite a source.
- Use a different sheet for each problem.
- Clearly write DRAFT on any draft page you hand in.
- **This is an alternative paper for people who proposed a problem for the actual competition.**

MOPWOP

May 13, 2016

Problem 1. An invertible 2×2-matrix M with real entries is called a *MOAWOA-matrix* if its inverse M^{-1} can be obtained by permuting the entries of M. Show that if M is a MOAWOA-matrix, then so is M^2.
An invertible 3×3-matrix M with real entries is called a *MOPWOP-matrix* if its inverse M^{-1} can be obtained by permuting the entries of M. Does the same conclusion hold for MOPWOP-matrices?

Problem 2. Suppose I and J are (real) open intervals of finite positive length, each interval not containing the other. Prove that there exists a $\lambda \neq 0$ such that $x \mapsto e^{\lambda x}$ maps I and J to intervals of equal length if and only if I and J have different lengths.

Problem 3. Consider n people that stand in a circle. Initially, each of them holds a red and a blue ball. In a turn, each person chooses one of his balls and hands it to the person on his right. Thus, after a turn everyone again holds two balls, but the distribution of colors may have changed. Determine all positive integers n for which there exists a sequence of turns that, from the described starting point, visits all possible color distributions of the $2n$ balls, without any color distribution occurring twice.

Problem 4. We consider sequences a_0, a_1, a_2, ... of real numbers that satisfy

$$a_n = 4a_{n-1}(1 - a_{n-1})$$

for all positive integers n. How many such sequences satisfy $a_{2016} = a_0$?

Problem 5. We are given N weights, with masses 1 kg, 2 kg, ..., N kg. We want to select at least two of these weights, such that their total mass equals the average mass of the other weights. Show that this is possible if and only if $N + 1$ is a square.

Problem 6. Decide whether there exists a function $f : \mathbb{R} \to \mathbb{Z}$ that is surjective on every infinite additive subgroup of $\mathbb{R}$.